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The combined Monte-Carlo and finite-volume method for radiation in a two-dimensional irregular geometry

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Abstract

This article proposes a combined procedure of the Monte-Carlo and finite-volume method (CMCFVM) for solving radiative heat transfer in absorbing, emitting, and isotropically scattering medium with an isolated boundary heat source. The conventional flux methods such as the finite volume and the discrete ordinate methods are known to be afflicted by the ray effects due to its own angular discretization. Thereby, a wiggling behavior in the solution used to take place. In order to tackle this problem, which is especially pronounced in a medium with an isolated heat source, the CMCFVM is suggested here and successfully applied to a two-dimensional irregular enclosure.

1. Introduction

In recent years, a study of radiative heat transfer in an irregular multidimensional geometry has received increasing attention with the development of more powerful computers. Its practical application resides in a need to accurately predict the thermal behavior in the heat exchanger and combustor. Therefore, several methods have been developed to solve the radiative transfer equation in the irregular geometry. Among others, there is the finite-volume method (FVM) for radiation [1,2] which has been successfully applied to several problems of body-fitted geometries [3,4]. In the meanwhile, the discrete ordinates method (DOM) was also extended to handle a body-fitted geometry, and its computational accuracy has been discussed [5]. Since the spatial domain is divided into a finite number of control volumes in the DOM and FVM, these methods have a computational compatibility with other control-volume based CFD approaches. While the DOM needs a quadrature set associated with directions and weights, the FVM has a flexibility in a selection of control angles preserving the conservation of radiant energy [2].

These flux methods are used to show a non-physical oscillation in solution on the boundary heat flux, which results from the ray effect. This wiggling behavior is caused by the finite discretization of the continuous control angle. Further details about this shortcoming in DOM is well described by Chai et al. [6] and some remedies are also suggested. To avoid this problem, many researchers have attempted to improve the solution by using an angular quadrature set as well as a spatial differencing scheme [6–10]. However, any concept, so far, could not totally correct

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Nomenclature

I	radiation intensity, W/(m^2 sr)
I_b	blackbody radiation intensity, W/(m^2 sr)
$\vec{n}_i$	unit normal vector at the control volume surface i
q^R	radiative heat flux, W/m^2
$\vec{r}$	position vector
$\vec{s}$	unit direction vector

Greek symbols

β_0	extinction coefficient, $\kappa_a + \sigma_s$, m^{-1}
ϵ_w	wall emissivity
κ_a	absorption coefficient, m^{-1}
σ_s	scattering coefficient, m^{-1}
Φ	scattering phase function, sr^{-1}
Ω	solid angle, sr
ω_0	single scattering albedo, σ_s/β_0

Superscripts

m, w	radiation direction

the ray effects as long as the angular discretization is used all over the domain. Therefore, another alternative for the conventional methods is needed to accurately predict the radiative heat transfer. If the Monte-Carlo method (MCM) [11,12] is employed to analyze the radiative heat transfer, an exact solution can be obtained within a statistical limit while almost eliminating the ray effect. But this method is based on the ray tracing technique so that it needs an enormously large computational time. Maryama and Aihara [13] presented a simple numerical method, Radiation Element Method by Ray Emission Model (REM), in order to analyze the radiative heat transfer in three-dimensional arbitrary configurations with isotropic scattering medium only. This method may be applied to analyse radiative heat transfer in a complex geometry using an unstructured grid such as the one used in finite element analysis. The ray effects may be minimized in this method, due to a large number of ray emissions and correction of view factors. However, when the isolated heat source is located in a medium the ray effects still exist, due to a finite polar angle discretization. Ramankutty and Crosbie [14] presented a modified discrete ordinate method as another alternative by applying a separate semi-analytical treatment for the intensities at the boundary, resulting in a reduction of the ray effects in a partially heated rectangular geometry. However, this method is not appropriate for an irregular complex geometry or for a problem with scattering medium, since a semi-analytical solution is very difficult to obtain. It is even more difficult for the analytical integration for an anisotropically scattering medium.

In this work, therefore, a combined Monte-Carlo and finite-volume method (CMCFVM) is proposed to deal with the ray effects in absorbing, emitting, and isotropically scattering medium which is surrounded by diffusely reflecting walls, when there is an isolated heat source on one of them. The computational efficiency for FVM on the inner domain as well as the exactness without ray effects for MCM at the boundary, is integrated here in CMCFVM. As will be shown in the following, the Monte-Carlo method is applied at the boundary instead of performing analytical integration that is suggested by Ramankutty and Crosbie [14] which is very difficult for complex geometry with the anisotropic scattering medium. The difference, therefore, from the previous studies, is that the CMCFVM can easily be implemented for an anisotropically as well as isotropically scattering problem with a complex irregular geometry, which is the key point in this study.

2. Formulations of the CMCFVM

The radiation intensity for an absorbing, emitting and scattering gray medium at any position, $\vec{r}$, along a path, $\vec{s}$ is governed by

$$\frac{\mathrm{d}I(\vec{r},\vec{s})}{\mathrm{d}s} = -\beta_0 I(\vec{r},\vec{s}) + \kappa_a I_b(\vec{r}) + \frac{\sigma_s}{4\pi}\int_{\Omega'=4\pi} I(\vec{r},\vec{s})\Phi(\vec{s}'\to\vec{s})\mathrm{d}\Omega' \tag{1}$$

where κ_a and σ_s are the absorption and scattering coefficients and $\beta_0 = \kappa_a + \sigma_s$ is the extinction coefficient. $\Phi(\vec{s}'\to\vec{s})$ is the scattering phase function for a radiation from incoming direction $\vec{s}'$ to scattered direction $\vec{s}$. The first term on the RHS in Eq. (1) represents an attenuation of radiation intensity due to absorption and out-scattering, while the last two terms account for an augmentation of intensity due to the gas emission as well as in-scattering. The boundary condition for a diffusely emitting and reflecting wall can be denoted by

$$I(\vec{r}_w, \vec{s}) = \epsilon_w I_b(\vec{r}_w) + \frac{1-\epsilon_w}{\pi}\int_{\vec{s}',\,\vec{n}_w<0} I(\vec{r}_w, \vec{s}')\,|\,\vec{s}'\cdot\vec{n}_w\,|\,d\Omega' \tag{2}$$

where ϵ_w is the wall emissivity and subscript w denotes the location of the wall, while $\vec{n}_w$ is the unit normal vector. The above equation illustrates that the leaving intensity is a summation of emitted and reflected intensities at the wall.

In order to attack the problem of ray effects in an enclosure comprising a partially heated diffuse wall, the CMCFVM is to be developed here. It is then applied to the analysis of the radiative heat transfer in an absorbing, emitting, and isotropically scattering medium for validation. To implement CMCFVM, above all, the intensity is divided into two parts, i.e., I^w and I^m, following the work by Modest [15].

$$I(\vec{r}, \vec{s}) = I^w(\vec{r}, \vec{s}) + I^m(\vec{r}, \vec{s}) \tag{3}$$

While I^w originates from the emission from the enclosure wall, I^m is traced back to the radiative source term in the medium.

A substitution of Eq. (3) into Eq (1) results in two radiative transfer equations for I^w and I^m. While I^w is governed by the following equation,

$$\frac{dI^w(\vec{r}, \vec{s})}{ds} = -\beta_0 I^w(\vec{r}, \vec{s}) + \frac{\sigma_s}{4\pi}\int_{\Omega'=4\pi} I^w(\vec{r}, \vec{s}')\Phi(\vec{s}'\to\vec{s})d\Omega' \tag{4}$$

with following boundary condition,

$$I^w(\vec{r}_w, \vec{s}) = \epsilon_w I_b(\vec{r}_w) + \frac{1-\epsilon_w}{\pi}\int_{\vec{s}'\cdot\vec{n}_w<0} I^w(\vec{r}_w, \vec{s}')\,|\,\vec{s}'\cdot\vec{n}_w\,|\,d\Omega' \tag{5}$$

the governing equation and boundary condition for I^m can be written as,

$$\frac{dI^m(\vec{r}, \vec{s})}{ds} = -\beta_0 I^m(\vec{r}, \vec{s}) + \kappa_a I_b(\vec{r}) + \frac{\sigma_s}{4\pi}\int_{\Omega'=4\pi} I^m(\vec{r}, \vec{s}')\Phi(\vec{s}'\to\vec{s})d\Omega' \tag{6}$$

$$I^m(\vec{r}_w, \vec{s}) = \frac{1-\epsilon_w}{\pi}\int_{\vec{s}'\cdot\vec{n}_w<0} I^m(\vec{r}_w, \vec{s}')\,|\,\vec{s}'\cdot\vec{n}_w\,|\,d\Omega' \tag{7}$$

For the case of modified DOM, Modest [15] and Ramankutty and Crosbie [14] divided the RTE into two equations by a different way as follows,

$$\frac{dI^w(\vec{r}, \vec{s})}{ds} = -\beta_0 I^w(\vec{r}, \vec{s}) \tag{8}$$

$$\frac{dI^m(\vec{r}, \vec{s})}{ds} = -\beta_0 I^m(\vec{r}, \vec{s}) + \kappa_a I_b(\vec{r}) + \frac{\sigma_s}{4\pi}\int_{\Omega'=4\pi} (I^w + I^m)(\vec{r}, \vec{s}')\Phi(\vec{s}'\to\vec{s})d\Omega' \tag{9}$$

An analytic solution of equation (8) is then obtained and substituted into Eq. (9). But when a scattering term exists, the analytical solution is not available so that Ramankutty and Crosbie [14] deal with the in-scattering term in Eq. (9) using a semi-analytical treatment and a numerical scheme. But this approach requires a tremendous effort for application to a complex irregular geometry or a problem with a special boundary condition. Furthermore, a treatment of the anisotropic scattering term introduces an additionally formidable complication.

In this study, while the radiative transfer equation for I^w, Eq. (4) is solved using the MCM rather than trying to find the analytic solution, the radiative transfer equation for I^m, Eq. (6) is solved by FVM. The reason for selecting the MCM in solving Eq. (4) is that it can be successfully applied to obtain the exact solution within the statistical limit without incurring ray effects. Moreover this method can easily be extended to a complex geometry with anisotropic scattering.

In order to implement FVM, Eq. (6) is integrated over a control volume and a control angle whereby the discretization equation is obtained [2,3]. Then, a step scheme is introduced such that a downstream facial intensity is set equal to the upstream nodal value [3]. It is not only a simple and convenient procedure, but also ensures positive intensity while not considering a geometric and directional complexity. Since the emission from the wall is taken into account in Eq. (5) which is solved by the MCM as stated above, here in FVM only a reflection term is considered as represented in Eq. (7). A derivation of the finite volume method for radiation has already been described and easily found in the literature [3,4] it is, therefore, recommended to refer to them for details.

Using the MCM, the radiation emitted from the wall is solved by tracing the trajectories of a certain number of particles. Although there are many excellent review papers [16,17], the procedure adopted in the work of Taniguchi et al. [11] is followed here, i.e. the READ (Radiant Energy Absorption Distribution) method. This method computes the exchange factors involved between elements to determine the radiative heat transfer. Once these factors are obtained for a specific problem, a different set of boundary conditions can be imposed without re-computing the exchange factors. Usually a very large number of bundles are chosen to simulate the radiation emitted from each wall element and then their trajectories are traced to estimate the heat flux or temperature. For the CMCFVM, the radiative heat flux at the enclosure wall can be obtained by superimposing each flux component which is calculated from the MCM and FVM, respectively.

3. Results and discussions

As shown in Fig. 1, the CMCFVM is now applied to a quadrilateral geometry with an isolated boundary heat source. This quadrilateral geometry has been widely used for testing the method for radiative heat transfer in an irregular geometry [12,18]. While the left wall is wholly or partially hot (1000 K) and black, the other walls are cold (300 K) and black with cold medium (300 K). The spatial grid used here is $(N_x \times N_y) = (21 \times 21)$ for all the cases presented below. In order to validate the present codes of the MCM and FVM, several preliminary calculations were performed for a two-dimensional rectangular geometry as well as the quadrilateral containing an absorbing, emitting medium. The results obtained were found to be in a very good agreement with the exact solutions. The present FVM has also been successfully applied to several problems [3,12]. Since the MCM requires a large number of energy bundles to produce a sufficiently accurate solution, the number of energy bundles was set to 1×10^7. The variance of the solution of the Monte-Carlo simulation could be estimated by carrying out several runs with different random number generators. The statistical error for the wall heat flux induced by the MCM was observed to be within 1%.

Fig. 2 represents the wall heat flux distribution along the periphery of the quadrilateral in which the absorbing, emitting, and isotropic scattering medium has an extinction coefficient, $\beta_0 = 1\ \mathrm{m}^{-1}$ and a scattering albedo, $\omega_0 = 0.5$, respectively. It is noted that only an upper half of the left wall is hot. The heat flux is non-dimensionalized by the blackbody emissive power of the hot wall. In Fig. 2(a), the solutions by FVM with a different number of control angles are compared to that of the MCM. The ray effects are clearly recognized by a wiggling behavior. It reveals that the ray effects are seen reduced by simply increasing the number of angular discretization in FVM, but they cannot be totally eliminated. Based on the results in Fig. 2(a), the presence of the ray effect is more conspicuous in such problems that have an isolated heat source for FVM. Therefore, the flux methods such as FVM and DOM need to be corrected for the problem with a localized heat source to deal with the ray effects.

The CMCFVM uses the concept of dividing the intensity into two parts. The wall heat flux is then obtained by adding the heat flux obtained by the MCM to that by FVM, which is distinctly illustrated in Fig. 2(b). It shows a remarkable accuracy of the CMCFVM that is almost comparable to the MCM with only 10–20% of computational time required by the MCM.

Fig. 3 shows the effect of the size of the isolated heat source on the wall heat flux along the periphery using the MCM, CMCFVM, and FVM. A whole, half, and quarter section heating of the left wall are considered. The isothermal medium (300 K) has an extinction coefficient, $\beta_0 = 1\ \mathrm{m}^{-1}$ and scattering albedo, $\omega_0 = 0.5$, respectively. As the heating size gets smaller, the ray effects are shown to increase when FVM is used. The CMCFVM is still observed to represent a very accurate result while keeping a computational efficiency.

The effects of scattering albedo on the non-dimensionalized wall heat flux are examined for the scattering albedo of 0.2, 0.5 and 0.8 in Fig. 4. As the scattering albedo decreases, absorption dominates the transport and more radiation is absorbed so that the wall heat flux gets smaller. While the results by FVM show a wiggling behavior as well as a significant inaccuracy, the CMCFVM shows none of the ray effects and inaccuracy in the figure.

Fig. 5 illustrates the effects of extinction coefficient on the wall heat flux when the scattering albedo is $\omega_0 = 0.5$. As the extinction coefficient gets smaller, the results by FVM deviate more from the Monte-Carlo solutions, since less energy is absorbed by the medium. Therefore, it is shown that the ray effects become more pronounced in the optically thinner case than the optically thicker case. In this case, the CMCFVM also yields very accurate results compared with those by the MCM.

In order to check the computational efficiency of the CMCFVM, the computation time required by FVM,

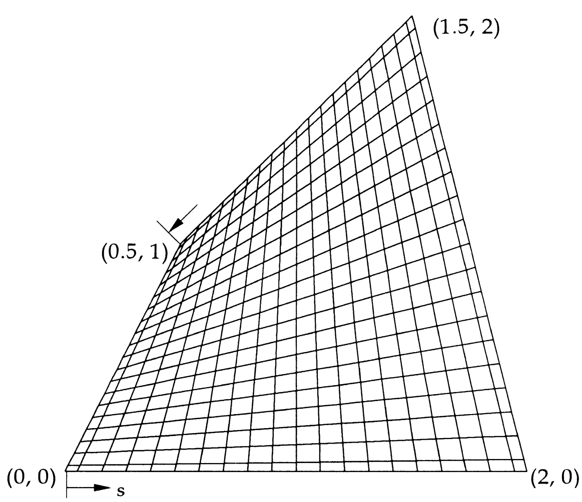

Fig. 1. Schematic of a quadrilateral with a body-fitted coordinate grid system.

CMCFVM and MCM on a IBM-PC machine with an Intel-450 processor are listed in Table 1 for various albedo. As the scattering albedo increases, the computation time required for FVM increases, since more iteration is needed to resolve the in-scattering as well as out-scattering term. Similarly, the MCM and CMCFVM also demand more time as the scattering albedo increases, since the energy bundles necessarily travel longer distances due to stronger scattering effects. While the CMCFVM requires longer computation time than FVM, which not only lacks in accuracy, but also induces the ray effects, the CMCFVM only needs about 23% of computation time spent by the MCM in generating accurate solutions comparable to those by MCM.

While a radiative heat transfer in an irregular

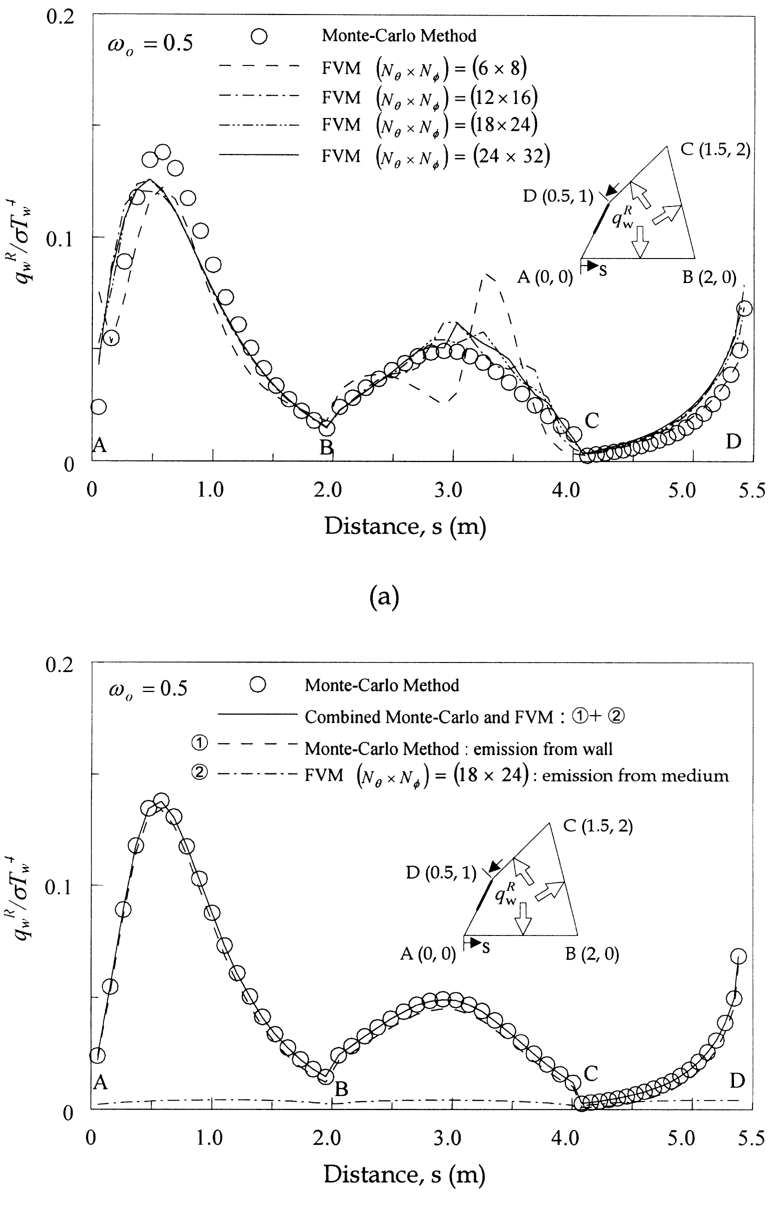

Fig. 2. Non-dimensional radiative heat flux along the periphery of the wall.

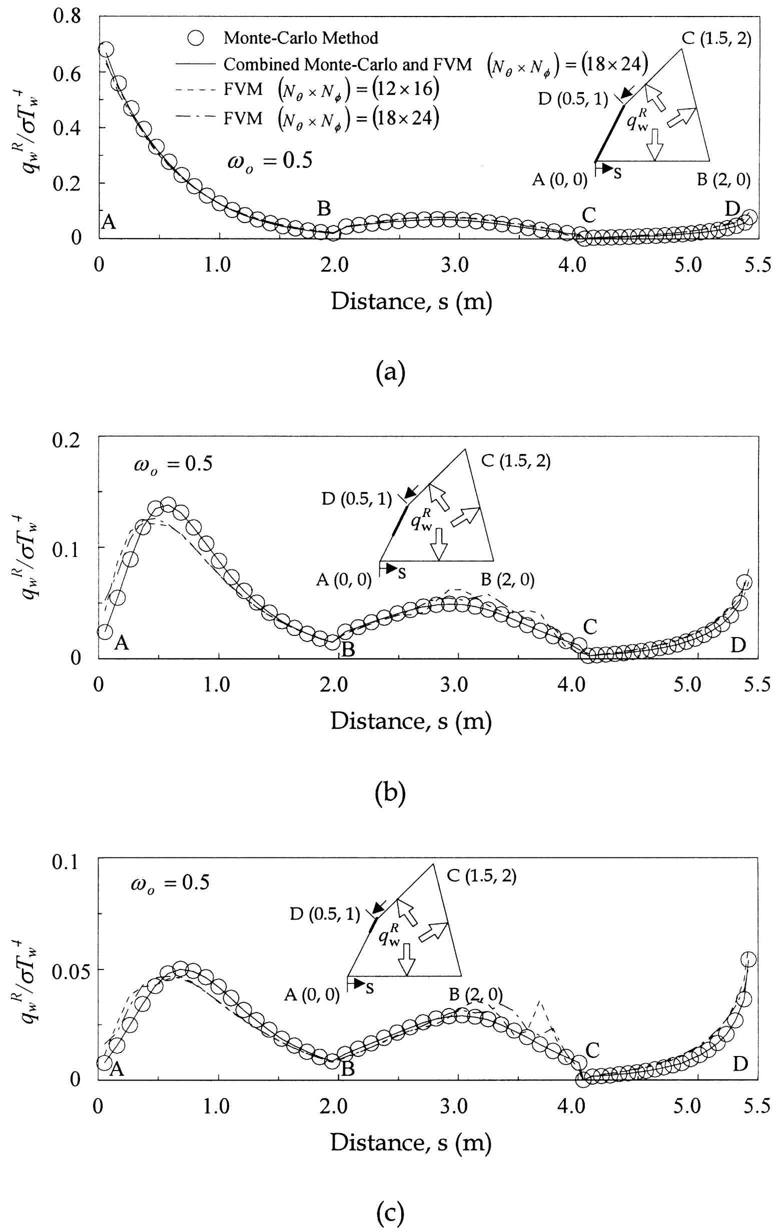

Fig. 3. Effect of the heater size on the non-dimensional radiative heat flux along the periphery of the wall: (a) whole left wall heating; (b) half wall heating; or (c) a quarter wall heating.

Table 1
Comparison of CPU time ($\beta_0 = 1\ \mathrm{m}^{-1}$)

		$(N_\theta \times N_\phi) = (18 \times 24)$		
	ω_0	MCM	FVM	CMCFVM
Quadrilateral	0.2	46,182	251	11,072
	0.5	56,460	257	13,477
	0.8	73,389	302	17,353

geometry with absorbing, emitting, and anisotropic scattering medium was already analyzed using the MCM by Parthasaranthy et al. [12], the FVM was also applied to an anisotropic scattering problem by Baek and Kim [19]. Since the CMCFVM makes simultaneously use of the merits of both the MCM and the FVM, it would be easily applied to other irregular complex geometries with a high computational efficiency even when accompanied by the anisotropic scattering effect.

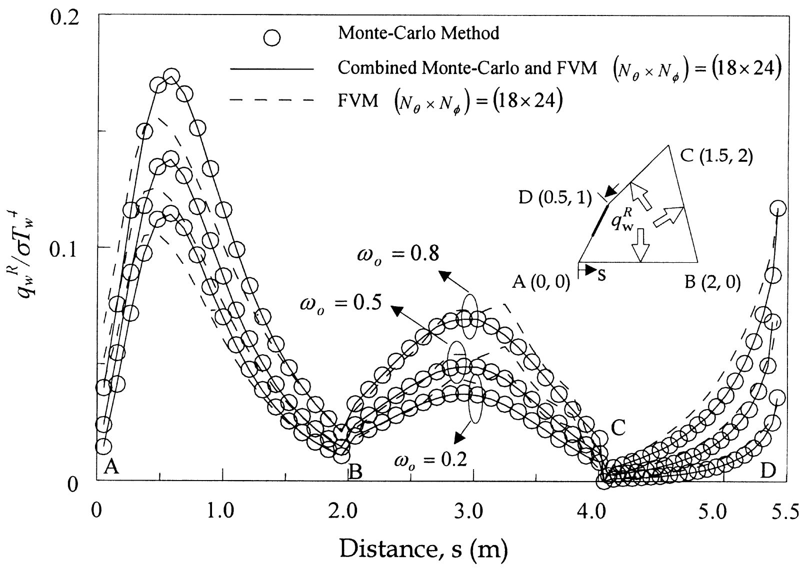

Fig. 4. Effect of scattering albedo on non-dimensional radiative heat flux along the periphery of the wall.

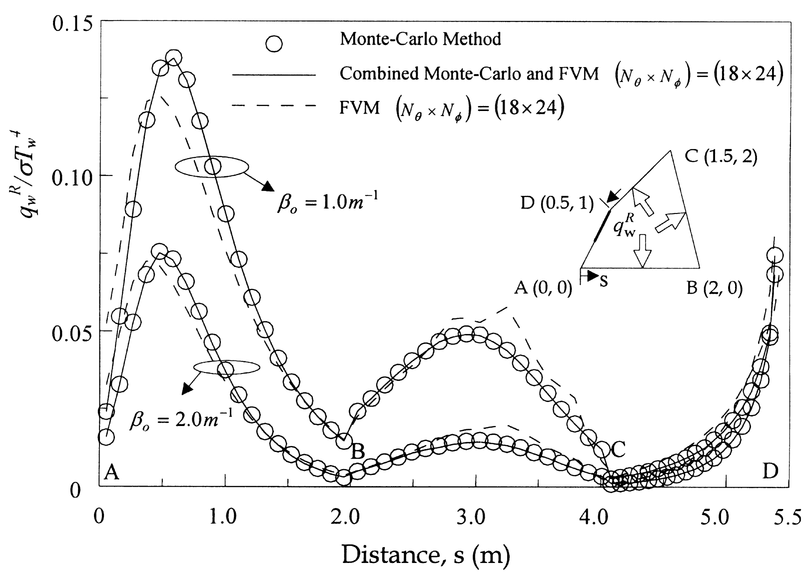

Fig. 5. Effect of extinction coefficient on non-dimensional radiative heat flux along the periphery of the wall.

4. Conclusions

The radiative heat transfer in an absorbing, emitting, and isotropic scattering medium with an isolated boundary heat source is analyzed using the MCM and the finite-volume method (FVM). Based on these results, in this study, however, the combined Monte-Carlo and finite-volume method (CMCFVM) has been proposed and is preferred by making use of the merits of the two methods. Thereby, it was clearly shown that the ray effects could be eliminated with a good computational efficiency in a two-dimensional irregular enclosure containing a scattering medium with an isolated heat source.

References

[1] E.H. Chui, G.D. Raithby, Computation of radiant heat transfer on a nonorthogonal mesh using the finite-volume method, Numerical Heat Transfer, Part B 23 (1993) 269–288.

[2] J.C. Chai, H.S. Lee, S.V. Patankar, Finite volume radiative heat transfer procedure for irregular geometries, AIAA Journal of Thermophysics and Heat Transfer 9 (3) (1995) 410–415.

[3] M.Y. Kim, S.W. Baek, Numerical analysis of conduction, convection, and radiation in a gradually expanding channel, Numerical Heat Transfer, Part A 29 (7) (1996) 725–740.

[4] S.W. Baek, M.Y. Kim, J.S. Kim, Nonorthogonal finite-volume solutions of radiative heat transfer in a three-dimensional enclosure, Numerical Heat Transfer, Part B 34 (4) (1998) 419–437.

[5] J.S. Liu, H.M. Shang, Y.S. Chen, T.S. Wang, Prediction of radiative transfer in general body-fitted coordinates, Numerical Heat Transfer, Part B 31 (1997) 423–439.

[6] J.C. Chai, H.S. Lee, S.V. Patankar, Ray effect and false scattering in the discrete ordinates method, Numerical Heat Transfer, Part B 24 (1993) 359–373.

[7] R. Koch, W. Krebs, S. Wittig, R. Viskanta, Discrete ordinate quadrature schemes for multidimensional radiative transfer, Journal of Quantitative Spectroscopy and Radiative Transfer 53 (4) (1995) 353–372.

[8] F. Liu, A. Becker, A. Pollard, Spatial differencing schemes of the discrete-ordinates method, Numerical Heat Transfer, Part B 30 (1996) 23–43.

[9] J.S. Truelove, Discrete–ordinate solutions of the radiation transport equation, ASME Journal of Heat Transfer 109 (1987) 1048–1051.

[10] A.S. Jamaluddin, P.J. Smith, Predicting radiative transfer in rectangular enclosures using the discrete ordinates method, Combustion Science and Technology 59 (1988) 321–340.

[11] H. Taniguchi, W.J. Yang, K. Kudo, H. Hayasaka, T. Fukuchi, I. Nakamachi, Monte Carlo method for radiative heat transfer analysis of general gas-particle enclosures, International Journal for Numerical Methods in Engineering 25 (1988) 581–592.

[12] G. Parthasarathy, H.S. Lee, J.C. Chai, S.V. Patankar, Monte Carlo solutions for radiative heat transfer in irregular two-dimensional geometries, ASME Journal of Heat Transfer 117 (1995) 792–794.

[13] S. Maruyama, T. Aihara, Radiation heat transfer of arbitrary three-dimensional absorbing, emitting and scattering media and specular and diffuse surfaces, ASME Journal of Heat Transfer 119 (1997) 129–136.

[14] M.A. Ramankutty, A.L. Crosbie, Modified discrete ordinates solution of radiative transfer in two-dimensional rectangular enclosures, Journal of Quantitative Spectroscopy and Radiative Transfer 57 (1) (1997) 107–140.

[15] F.M. Modest, Modified differential approximation for radiative transfer in general three-dimensional media, AIAA Journal of Thermophysics and Heat Transfer 3 (3) (1989) 283–288.

[16] J.P. Howell, Application of Monte Carlo method to heat transfer problems, in: J.P. Hartnett, T.F. Irvine (Eds.), Advances in Heat Transfer, vol. 5, Academic Press, New York, 1968, pp. 1–54.

[17] A. Haji Sheikh, Monte Carlo methods, in: W.J. Minkowycz, E.M. Sparrow, R.H. Pletcher, G.E. Schneider (Eds.), Handbook of Numerical Heat Transfer, Wiley, New York, 1988.

[18] K.B. Cheong, T.H. Song, An alternative discrete ordinates method with interpolation and source differencing for two-dimensional radiative transfer problems, Numerical Heat Transfer, Part B 32 (1997) 107–125.

[19] S.W. Baek, M.Y. Kim, Analysis of radiative heating of a rocker plume base with the finite-volume method, International Journal of Heat and Mass Transfer 40 (7) (1997) 1501–1508.